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THESIS

ANGULAR TRACKING ERROR IN A PHASE
COMPARISON MONOPULSE TRACKING RADAR,
A CRITICAL REVIEW AND EXTENSION OF THE
PHASE FRONT DISTORTION APPROACH

by

Sopon Bumroongpol

December 1984

Thesis Advisor:

H. M. Lee

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Angular Tracking Error in a Phase Comparison Monopulse Tracking
Radar, A Critical Review and Extension of
the Phase Front Distortion Approach

by

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Lieutenant Commander, Royal Thai Navy, Thailand
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Submitted in partial fulfillment of the
requirements for the degree of

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December 1984

ABSTRACT

This thesis studies the inherent angular errors of a phase comparison monopulse system used for tracking a complex target. The phase compensation equation is utilized in justifying Howard's hypothesis on the relationship between the phase front distortion of the scattered wave from a complex target and angular tracking errors, in extending this hypothesis to closer ranges to the target, and in determining the limitations of this hypothesis. Through the phase compensation equation, global errors are demonstrated. A Local error bound is also determined for the tracking of a two element target. These new results are not predicted by Howard's hypothesis.

TABLE OF CCNTENTS

I.	INTRODUCTION	7
	A. RADAR FOR TRACKING	7
	B. NOISE IN TRACKING RADARS	12
	C. HOWARD'S ASSUMPTION	15
	D. PURPOSE OF THE THESIS	16
II.	THEORETICAL BACKGROUND AND RESULTS FROM SCATTERING THEORY	17
	A. RADAR CROSS SECTION	17
	B. THE ELECTROMAGNETIC SCATTERING	18
III.	PHASE FRONT DISTORTION AND THE ANGULAR TRACKING ERROR OF A PHASE COMPARISON MONOPULSE TRACKING SYSTEM	23
	A. RELATION BETWEEN ANGLE ERROR AND PHASE FRONT	23
	B. PHASE FRONT OF THE SCATTERED FIELD FROM A TARGET AND THE ANGULAR TRACKING ERROR.	24
	C. PHASE COMPENSATION EQUATION AND THE ANGULAR TRACKING ERRCRS	30
IV.	AZIMUTHAL ANGULAR ERROR FOR A TWO-ELEMENT TARGET	36
V.	CONCLUSION AND REMARKS	39
	LIST OF REFERENCES	40
	INITIAL DISTRIBUTION LIST	41

LIST OF FIGURES

1.1	Conical-Scan Tracking	8
1.2	Monopulse Antenna Patterns and Error Signal. . . .	11
1.3	Wavefront Phase Relationships in Phase Comp. Monopulse Radar	13
2.1	Bistatic Case	19
2.2	Monostatic Case	22
3.1	Coordinates of a Simplified Phase Comparison Tracking System	26
3.2	Phase Fronts and A Phase Comparison Tracking System	29
3.3	Phase Compensation	32
3.4	Local and Global Angular Tracking Errors	34
3.5	Max. Local Angular Error of a Phase Comparison System	35
4.1	Angular Tracking Error across a Singularity of Phase Distortion	37
4.2	Maximal Local Error, First and Second Global Error	38

I. INTRODUCTION

A. RADAR FOR TRACKING

A tracking-radar system measures the coordinates of a target and provides data which may be used to determine the target path and to predict its future position. All or only part of the available radar data such as range, elevation angle, azimuth angle and doppler frequency shift may be used in predicting future target position; that is, a radar might track in range, in angle, in doppler, or with any combination. Almost any radar can be considered a tracking radar provided its output information is processed properly. But it is the method by which angle tracking is accomplished that distinguishes what is normally considered a tracking radar from any other radar. It is also necessary to distinguish between a continuous tracking radar and a track-while-scan (TWS) radar. The former supplied continuous tracking data on a particular target, while the track-while-scan supplies sampled data on one or more targets. In general, the continuous tracking radar and the TWS radar employ different types of equipment and serve different purposes.

For TWS, the track of a target can be determined with a surveillance radar from the coordinates of the target as measured from scan to scan. The quality of such a track will depend on the time between observations, the locating accuracy of each observation, and the number of extraneous targets that might be present in the vicinity of the tracked target. All of these, including prediction and estimation, are usually accomplished by using a specific computer.

For continuous tracking radars, a popular type is the conical scan radar, as shown in figure 1.1 The angle between

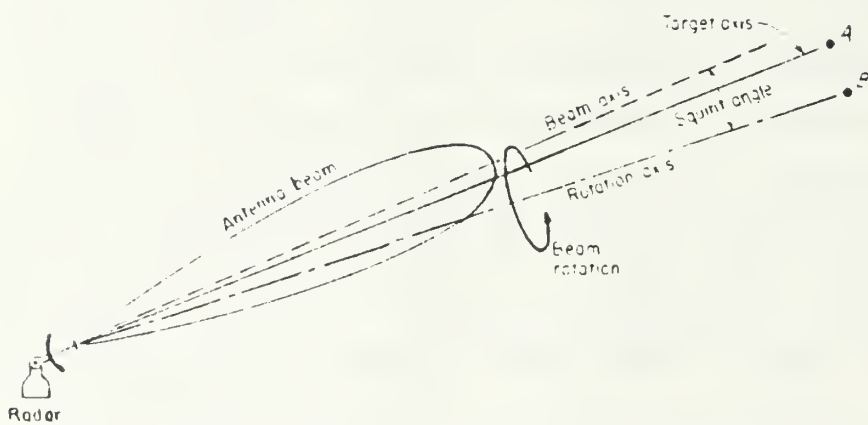


Figure 1.1 Conical-Scan Tracking.

the axis of rotation (which is usually the axis of the antenna reflector) and the axis of the antenna beam is called the squint angle. Consider a target at position A. The echo signal will be modulated at a frequency equal to the rotation frequency of the beam. The amplitude of the echo-signal modulation, called the angle error signal, will depend upon the shape of the antenna pattern, the squint angle, and the angle between the target line of sight and the rotation axis. The phase of the modulation depends on the angle between the target and the rotation axis. The conical scan modulation is extracted from the echo signal and applied to a servo-control system which continually positions the antenna on the target. Note that two servos are required because the tracking problem is two-dimensional. Both the rectangular and polar tracking coordinates may be used. When the antenna is on target, and if the target is located at B of figure 1.1, then the line of sight to the target and the rotation axis coincide, and the conical-scan amplitude modulation is zero. The conical-scan tracking radar requires information from a number of pulses in order to extract the angle-error signal. In the time interval during which a conical-scan measurement is made, the train of echo pulses must contain no amplitude-modulation components other than the modulation produced by scanning. If the echo pulse-train contains additional modulation components caused, for example, by a fluctuating target cross section, the tracking accuracy might be degraded. This can be especially severe if the spectral content of the fluctuation is strong at or near the conical-scan frequency or the sequential-lobing rate. The effect of the fluctuating echo can be sufficiently serious in some applications to severely limit the accuracy of those tracking radars which require many pulses to be processed before the error signal can be extracted.

The monopulse tracking technique is superior to the conical-scan because all information necessary for the determination of the angular error is obtained through a single pulse. Since the target appears stationary for a pulse duration, pulse to pulse variations of the echo signal due to target motion, which limit the performance of a conical-scan radar, have no effect on a monopulse radar.

The amplitude-comparison monopulse employs two identical but slightly offset antennas (figure 1.2 a) to provide the angular error in one coordinate. The two overlapping antenna beams may be generated with a single reflector or with a lens antenna illuminated by two adjacent feeds. (A cluster of four feeds may be used if both elevation and azimuth error signals are desired.) The sum of the two antenna patterns of figure 1.2 (a) is shown in figure 1.2 (b), and the difference in figure 1.2 (c). The sum pattern is used for transmission, while both the sum pattern and the difference pattern are used on reception. The signal received with the difference pattern provides the magnitude of the angle error. The sum signal provides the range measurement and is also used as a reference to extract the sign of the error signal. They are amplified separately and combined in a phase-sensitive detector to produce the error signal characteristic shown in figure 1.2 (d), The useful region is on the linear portion of the curve.

In a phase comparison monopulse radar, the angle of arrival (in either the azimuth or the elevation direction of the radar) is determined by comparing the phase difference between the signals from two separate antennas. Unlike the antennas of amplitude comparison trackers, those used in phase comparison systems are not offset from the axis. The individual boresight axes of the antennas are parallel, causing the (far-field) radiation to illuminate the same volume in space. The amplitudes of the target echo signal

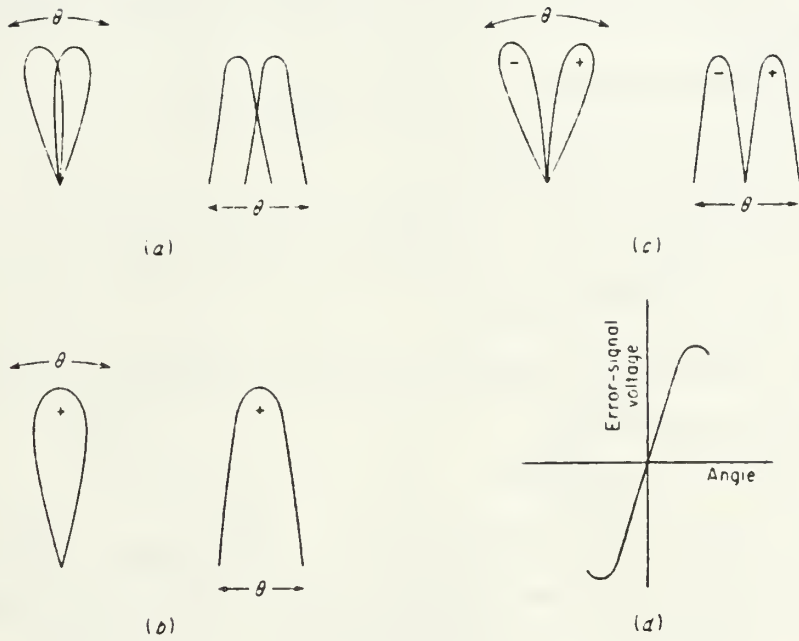


Figure 1.2 Monopulse Antenna Patterns and Error Signal
 Left-Hand Diagrams in (a-c) are in Polar -
 Coordinate: Right-Hand Diagrams are in Rec-
 tangular Coordinates. (a) Overlapping Ante-
 nna Patterns; (b) Sum Pattern; (c) Differe-
 nce Pattern; (d) Product (error) Signal.

are essentially the same from each antenna beam, but the phases are different. A tracking radar which operates with phase information is similar to an active interferometer and might be called an "interferometer radar". In figure 1.3 two antennas are shown separated by a distance d . The distance to the target is R and is assumed large compared with the antenna separation d . The line of sight to the target makes an angle θ to the perpendicular bisector of the line joining the two antennas.

The distance from antenna B to the target is

$$R_B \doteq R + d \cdot \sin \theta / 2$$

and the distance from antenna A to the target is

$$R_A \doteq R - d \cdot \sin \theta / 2$$

The phase difference between the echo signals in the two antennas is approximately

$$\Delta \phi = 2\pi d \cdot \sin \theta / \lambda$$

For small angles where $\sin \theta \doteq \theta$, the phase difference is a linear function of the angular error and may be used to position the antenna via a servo-control loop. [Ref. 1]

B. NOISE IN TRACKING RADARS

The target-noise or target-scintillation is due to the physical complexity and maneuvers of the target. Because of the size of the target compared to the wavelengths being used, the amplitudes of radar echoes from moving targets of practical interest such as an aircraft will fluctuate widely and rapidly as target aspect changes. In general, there will be rapid fluctuation caused by pitching and yawing of the airframe about the line of flight, and slow variations caused by change in the average aspect of the air frame relative to the radar.

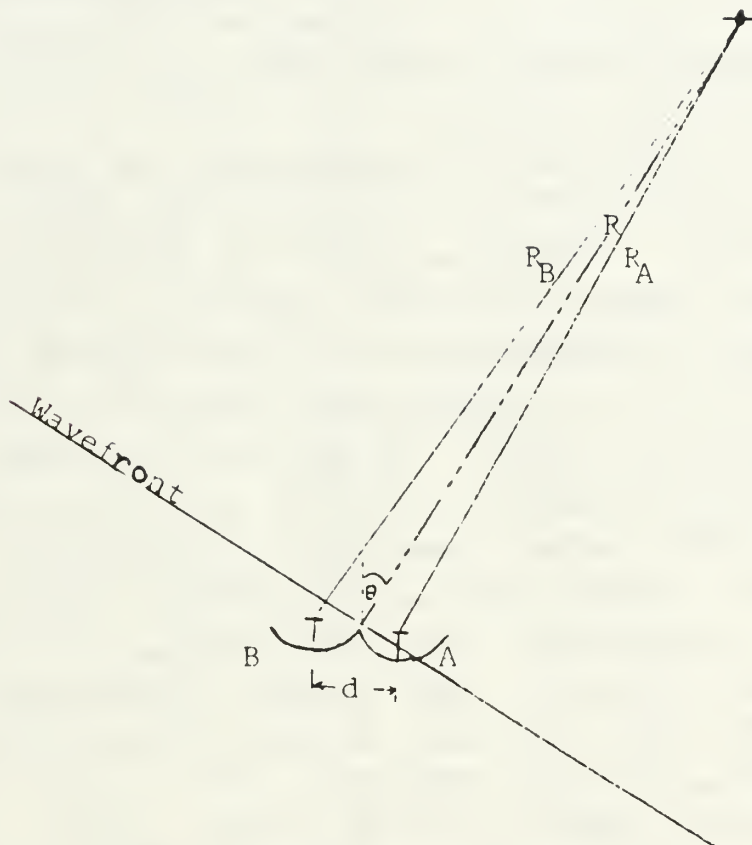


Figure 1.3 Wavefront Phase Relationships in Phase Comparison Monopulse Radar.

The noise sources which contribute to tracking noise may be separated into four components, i.e.,

1. Servo noise is the error of the tracking servomechanism which results from backlash and compliance in the gears, shafts, and structures of the mount. The magnitude of this noise is essentially independent of the target and will thus be independent of range.
2. Receiver noise is the effect on the tracking accuracy of the radar due to thermal noise generated in the receiver and any spurious hum which may be picked up by the circuitry.
3. Angle noise (angle scintillation or glint) is the tracking error introduced into the radar by variations in the apparent angle of arrival of the echo from a complex target of finite size. This effect is caused by variations in the phase front of the radiation from a multiple-point target as the target changes its aspect. The magnitude of angle noise is inversely proportional to the range of the target.
4. Amplitude noise (amplitude scintillation) is the effect on the radar accuracy of the fluctuations in the amplitude of the signal returned by the target. These fluctuations are caused by any change in aspect of the target and must be taken to include propeller rotation and skin vibration.

The first two (No.1,2) of these components originate in the radar itself. The second two components (No.3,4) are called target noise.

Angle noise is the most important error at medium to close range. [Ref.2]

Superiority of monopulse tracking radar over conical scan tracking radar is based on the fact that a target appears stationary over a pulse duration but varying from pulse to pulse. Monopulse tracking radar still encounters

angle errors for each pulse. This may result in increased pulse to pulse variations in predicted target locations and causes problems in the servo system. This target noise should be dealt with through proper choice of the servo-bandwidth which determines the sensitivity of the servo system.

C. HOWARD'S ASSUMPTION

The "slope" of the phase front of the echo signal from a complex target of finite size is claimed by Howard [Ref.4] to be identical to the angular errors caused in a tracking radar by angular scintillation or target angle noise. No definition on "slope" was given, however, and no proof of the statement was provided. Since phase comparison tracking systems are essentially phase front positioning devices, it can be argued that the target angle noise is determined by the echo signal due to the distortion of its phase front from being a sphere. Howard and Dunn [Ref.5] further proposed, by erroneously assuming that an antenna is a power measuring device instead of a field strength probe, that deviations in the direction of the echo signal is given by the Poynting vector. They argued that the angular error from a complex target can be so large that the apparent source falls many target spans away from the actual target location. Howard further claimed without proof that the relationship between phase front distortion and angular tracking error also applies to the amplitude comparison tracking of a target and to the search radars. These claims have been proved to be false. It has been found that angular tracking error studies based on Howard's assumption could lead to exaggerated error estimates

D. PURPOSE OF THE THESIS

In this thesis a detailed study is carried out to demonstrate the true relationship between the phase of the scattered field and the angular tracking error in a phase comparison monopulse tracking radar.

II. THEORETICAL BACKGROUND AND RESULTS FROM SCATTERING THEORY

A. RADAR CROSS SECTION

For most practical situations there is relative motion between a radar and the target being tracked. The radar "sees" a dynamic cross section. Considerable effort has been expended in investigating the statistics of dynamic cross sections, particularly in the case of aircraft. It can be shown that for a target consisting essentially of a large number of point scatterers in random relative motion, the radar cross section will have an exponential probability density. For a relatively complex target, such as an aircraft at high frequencies, experimental results agree well with this conclusion (see detail in Ref.4). That is we can use the static cross section in our calculation instead of using dynamic cross section.

However, for most of the calculations and analysis of this thesis the target may be assumed to be stationary, because a monopulse tracking radar is used. That is, the radar cross section is a static cross section within a pulse duration. There exists a wide variety of analytical techniques for computing radar cross sections. Some of these lead, in principle, to exact solutions, although approximations are usually required to obtain numerical results even from what was originally an exact formulation. A typical example of this type of technique is the formulation of a scattering problem as a boundary-value problem with the resultant formally exact solution appearing as an infinite series. To obtain numerical results, the sum of this series must usually be approximated in some fashion.

Other techniques involve various simplifying assumptions to arrive at the basic formulation so that even if the mathematical analysis can be carried through exactly, only an approximate solution is obtained. A typical example of this type is the physical optics technique for which it is generally true that even if the required integration can be performed exactly, only an approximate solution is obtained. The definition of radar cross section is as follows:

$$\sigma = \lim_{R \rightarrow \infty} 4\pi R^2 |E_s/E_i|^2$$

where E_s is the electric field intensity scattered from the target and observed at a distance R from the target, and E_i is the field intensity incident upon the target. The incident field is assumed to be a uniform plane wave, and the limit of E_s as $R \rightarrow \infty$ is taken to ensure that the true far-field value of σ is measured. In practice, it is usually considered sufficient if R is greater than about $2D^2/\lambda$, where D is the largest dimension of the target in any direction transverse to the direction from which E_s is observed and λ is the radar wavelength. It should be noted, however, that for some purposes the $2D^2/\lambda$ criterion is inadequate and the recommended far-field distance may be several times as great.[Ref.5]

Note that, the radar cross section also depends upon the frequency, shape and composition of the scattering object.

B. THE ELECTROMAGNETIC SCATTERING

In general, for bistatic radar the scenario is shown in Figure 2.1

where

\hat{i} is the direction of the propagation vector of the incident plane wave.

\vec{E}_i, \vec{H}_i , are the fields of the incident wave.

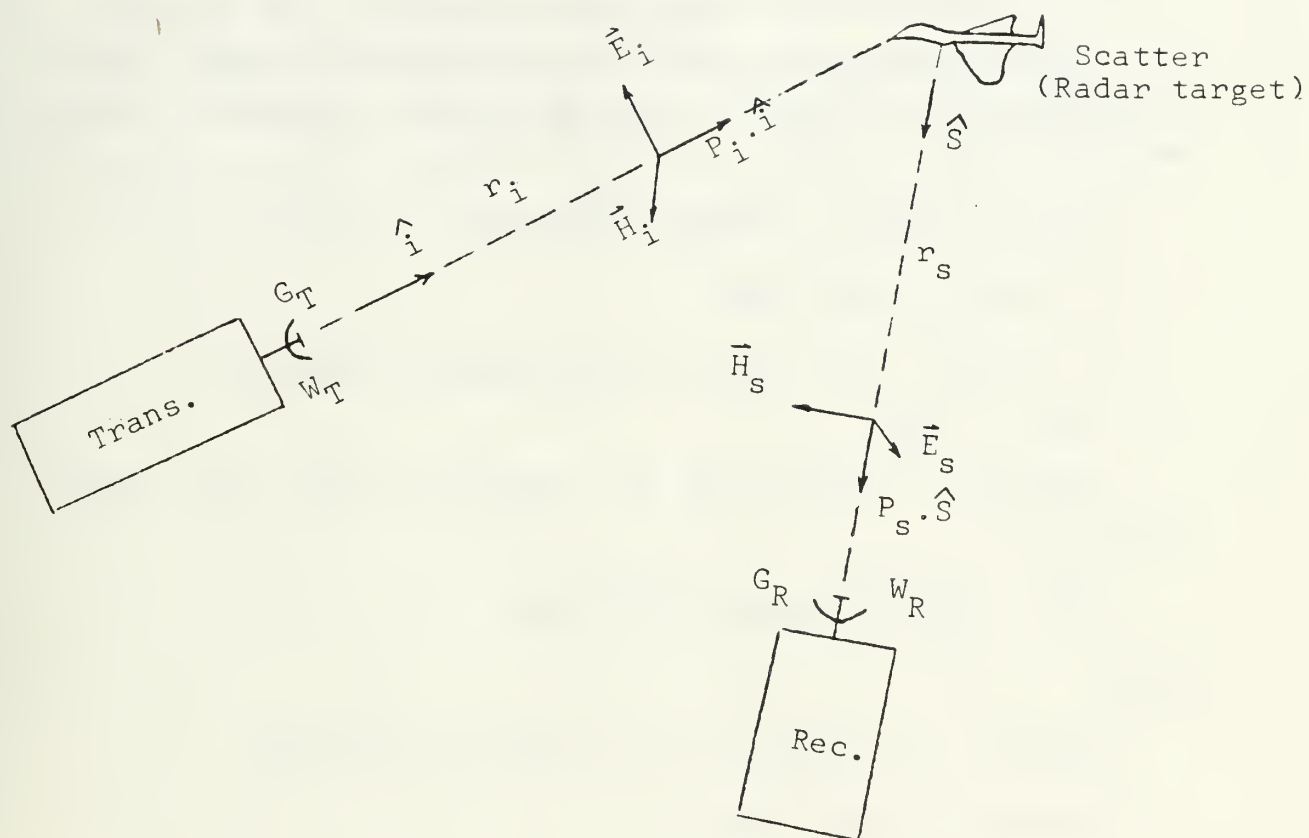


Figure 2.1 Bistatic Case.

r_i is the distance from the transmitter to the scatterer.

\hat{s} is the radial direction unit vector from the scatterer to the receiver

\bar{E}_s, \bar{H}_s , are the far fields of the scattered wave.

r_s is the distance from the scatterer to the receiver.

The scatterer is a metallic object with induced surface currents \bar{J}_s , which can be solved for example, by the magnetic field integral equation (MFIE).

$$\bar{J}_s(\bar{r}) = 2\hat{n} \times \bar{H}_i(\bar{r}) + \hat{n} \times \iint_S \bar{J}_s(\bar{r}') \times \nabla G \cdot d\bar{s}$$

$$G = \{\exp.(-jkR)\} / 2\pi R$$

$$R = |\bar{r} - \bar{r}'|, \text{ for } \bar{r} \text{ and } \bar{r}' \text{ both points on surface}$$

$$\text{and } k = 2\pi/\lambda.$$

Scattered fields can be obtained from the surface current. If

$$\bar{f}(\hat{s}) = \iint_S \bar{J}_s(\bar{r}') \cdot \{\exp. (+jk\hat{s}\bar{r}')\} \cdot d\bar{s}$$

then

$$\bar{E}_s(\bar{r}_s) = \{-jk\eta_0/4\pi r_s\} \cdot \{\exp.(-jkr_s)\} \cdot \{\bar{f} - (\bar{f} \cdot \hat{s})\hat{s}\}$$

$$\bar{H}_s(\bar{r}_s) = (1/\eta_0) \hat{s} \times \bar{E}_s(\bar{r}_s)$$

Linearity results in \bar{J}_s being proportional to \bar{E}_i , and a "complex scattering amplitude function" can be defined as:

$$\bar{S}(\hat{i}, \hat{s}) = \{-jk\eta_0/4\pi E_i\} \cdot \{\bar{f} - (\bar{f} \cdot \hat{s})\hat{s}\}$$

then

$$\bar{E}_s(\bar{r}_s) = E_i \bar{S}(\hat{i}, \hat{s}) \cdot \{\exp.(-jkr_s)\} / r_s$$

where $\bar{S}(\hat{i}, \hat{s})$ is polarization dependent.

More generally, polarization effects can be included in the "scattering matrix" as:

$$\begin{bmatrix} E_{S1} \\ E_{S2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} E_{i1} \\ E_{i2} \end{bmatrix} \frac{e^{-jk r_s}}{r_s}$$

$$\text{or, } \bar{E}_s = \tilde{S}(\hat{i}, \hat{s}) \cdot \bar{E}_i \cdot \{\exp.(-jkr_s)\} / r_s$$

The subscripts 1 and 2 represent polarizations for both the incident and the scattered fields.

For monostatic scattering, the scenario is shown in figure 2.2 Usually the same antenna is used for transmitting and receiving in a radar.

For the monostatic case, $r_i = r_s = r$.

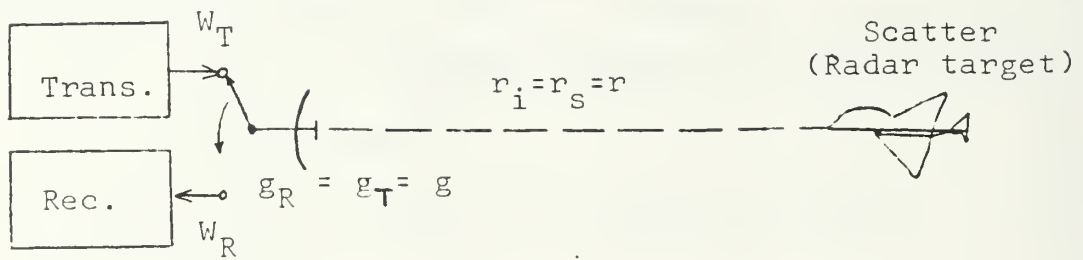


Figure 2.2 Monostatic Case.

III. PHASE FRONT DISTORTION AND THE ANGULAR TRACKING ERROR OF A PHASE COMPARISON MONOPULSE TRACKING SYSTEM

A. RELATION BETWEEN ANGLE ERROR AND PHASE FRONT

The phase compensation equation is found to describe the mechanism with which the direction of the tracking axis of a phase comparison tracking system is determined [Ref.7]. Howard's phase front distortion technique is shown to be applicable to such a system and is an approximation to the phase compensation equation. In what follows, global angular tracking errors are demonstrated and an upper bound to the local tracking error is given. The antenna separation is recognized to be the essential parameter in the determination of both kinds of errors.

Using the far field approximation of the scattered fields from a two-element target, Howard [Ref.4] reasoned that the angular error of a tracking radar is determined by the distortion of the phase front of the scattered field from a target. The significance of this observation stems from the fact that it relates the angle noise of a tracking radar directly to the scattering characteristics of the target. Hence the parameter of the antenna system in a tracking radar need not be considered and studies in angle noises are greatly simplified.

A recent investigation [Ref.6] revealed that for an amplitude comparison system, the tracking radar can be right on target yet Howard's phase front distortion technique predicts an infinite error [Ref.1]. This brings about the question of whether angle noise predictions based on Howard's assumption have been exaggerated for some system while inapplicable to others. In this thesis, the relation

between the phase front of the scattered field and the angular tracking error of a phase comparison tracking system is examined carefully. The validity and restrictions of this relationship are established for local tracking errors. Global tracking errors are found to exist when the system is on a wrong track while no error signal is generated. In a companion work the angular tracking error of an amplitude comparison system are studied [Ref.8,9].

B. PHASE FRONT OF THE SCATTERED FIELD FROM A TARGET AND THE ANGULAR TRACKING ERROR.

From the three components of the scattered electric field, an antenna will pick up only a linear combination of them. The received field of concern for an antenna located at (r, θ, ϕ) can be written as :

$$F(r, \theta, \phi) \{ \exp[iD(r, \theta, \phi)] \} \{ \exp(ikr) \} / r \quad (1)$$

where $k = 2\pi/\lambda$ is the wave number, λ is the wavelength and $F(r, \theta, \phi) \geq 0$. The origin of the spherical coordinate system is located at the target. In the limit that r approaches infinity,

$$\begin{aligned} F(r, \theta, \phi) &\longrightarrow f(\theta, \phi) \\ D(r, \theta, \phi) &\longrightarrow \delta(\theta, \phi) \end{aligned} \quad (2)$$

Assume the incident field to be of unit strength and zero phase at the origin. Then $4\pi f^2(\theta, \phi)$ is the cross section of the target and $\delta(\theta, \phi)$ is the phase shift of the target.

Note that

$$f(\theta, \phi) \{ \exp[i\delta(\theta, \phi)] \} \{ \exp(ikr) \} / r$$

is the far field expression for equation (1) and $F(r, \theta, \phi)$ and $D(r, \theta, \phi)$ are slowly varying functions in r . For large r the phase of the scattered wave is :

$$\phi(r, \theta, \phi) = kr + D(r, \theta, \phi) \quad (3)$$

and a phase front is a surface on which $\phi(r, \theta, \phi)$ is constant. Thus $D(r, \theta, \phi)$ is the phase distortion which keeps the phase front from being a sphere centered at the target.

For simplicity, consider only the case when there is no error in the elevation angle. Only the plane containing the center of the target (i.e. origin of the spherical coordinate) and the pair of effective centers of the antennas measuring azimuth angle has to be considered. It can be modeled as a system of only two antennas.

As shown in figure 3.1, the angle ϕ_e from the negative radial direction $-\hat{r}_T$ to the tracking axis pointing direction \hat{t} (the caps above the characters denote unit vectors) is the angular tracking error. The antennas A and B, centered at $(r_A, \frac{\pi}{2}, \phi_A)$ and $(r_B, \frac{\pi}{2}, \phi_B)$ respectively, read the phases ϕ_A and ϕ_B according to equation (3) :

$$\phi_A = kr_A + D(r_A, \frac{\pi}{2}, \phi_A)$$

$$\phi_B = kr_B + D(r_B, \frac{\pi}{2}, \phi_B)$$

Note that this statement is based on the assumption that the angle extended by the target is negligibly small compared to the beam width of each of the antennas.

An error signal is generated in the tracking system if ϕ_A differs from ϕ_B . The tracking axis is conditioned to point to a direction so as to null the error signal. Therefore the tracking error is determined by the equation :

$$k(r_A - r_B) = D(r_B, \frac{\pi}{2}, \phi_B) - D(r_A, \frac{\pi}{2}, \phi_A) \quad (4)$$

The quantities r_A , r_B , ϕ_A , ϕ_B can be expressed in terms of r_T , ϕ_T , d , ϕ_e , through the following equations :

$$r_A^2 = r_T^2 + r_T d \sin \phi_e + d^2/4 \quad (5)$$

$$r_B^2 = r_T^2 - r_T d \sin \phi_e + d^2/4 \quad (6)$$

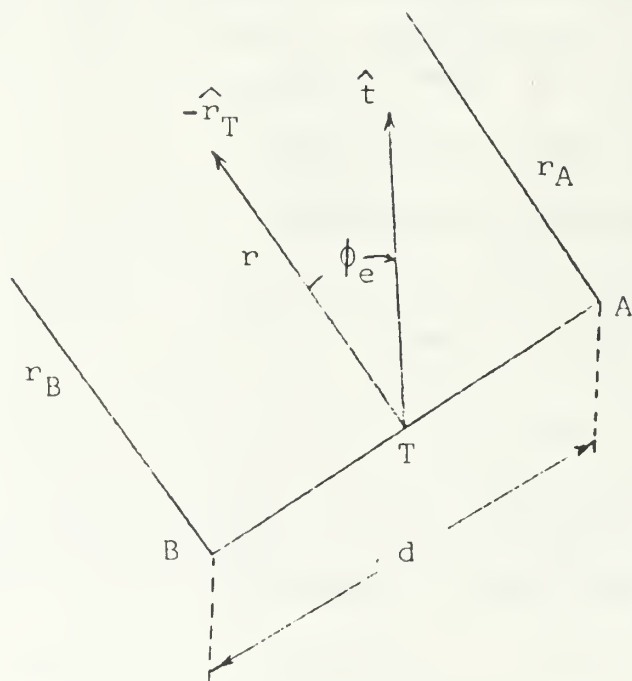
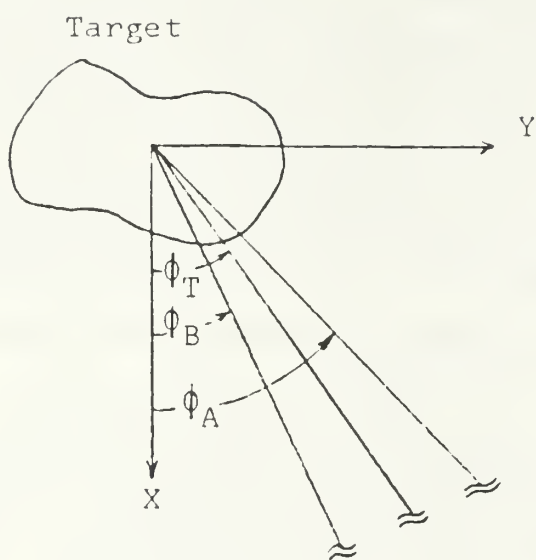


Figure 3.1 Coordinates of a Simplified Phase Comparison Tracking System and Its Target.

$$r_A \cdot \sin(\phi_e - \phi_T + \phi_A) = r_T \cdot \sin \phi_e + d/2 \quad (7)$$

$$r_B \cdot \sin(\phi_e - \phi_T + \phi_B) = r_T \cdot \sin \phi_e - d/2 \quad (8)$$

Where d is the separation between the antennas and $(r_T, \frac{\pi}{2}, \phi_T)$ is the midpoint T between the antennas A and B. Together with equations (5) to (8), equation (4) gives ϕ_e in term of r , ϕ_T , and d

With the assumption that $r_T \gg d$, to the lowest order in the ratio d/r_T equations (5) and (6) lead to :

$$r_A - r_B = d \cdot \sin \phi_e$$

equations (7) and (8) lead to :

$$\begin{aligned} \sin(\phi_e - \phi_T + \phi_A) - \sin(\phi_e - \phi_T + \phi_B) &= 2 \cdot \sin\left(\frac{\phi_A - \phi_B}{2}\right) \cdot \cos\left(\phi_e + \frac{\phi_A + \phi_B}{2} - \phi_T\right) \\ &= (\phi_A - \phi_B) \cos \phi_e \\ &= (1/r_A - 1/r_B) r_T \sin \phi_e + (1/r_A - 1/r_B) d/2 \\ &= d \cdot \cos^2 \phi_e \end{aligned}$$

$$\text{or} \quad \phi_A - \phi_B = d \cdot \cos \phi_e$$

To the same order, and assuming that $D(r, \frac{\pi}{2}, \phi)$ varies smoothly over the region between the antennas, the error signal nulling condition as given by equation (4) reduces to :

$$\begin{aligned} k(r_A - r_B) &= kd \cdot \sin \phi_e \\ &= D(r_B, \frac{\pi}{2}, \phi_B) - D(r_A, \frac{\pi}{2}, \phi_A) \\ &= (r_B - r_A) \frac{\partial}{\partial r} D(r_T, \frac{\pi}{2}, \phi_T) + (\phi_B - \phi_A) \frac{\partial}{\partial \phi} D(r_T, \frac{\pi}{2}, \phi_T) \\ &= -d \left\{ \sin \phi_e \cdot \frac{\partial}{\partial r} D(r_T, \frac{\pi}{2}, \phi_T) + \cos \phi_e (1/r) \frac{\partial}{\partial \phi} D(r_T, \frac{\pi}{2}, \phi_T) \right\} \quad (9) \end{aligned}$$

or

$$\tan \phi_e = - \left\{ (1/kr) \frac{\partial}{\partial \phi} D(r_T, \frac{\pi}{2}, \phi_T) \right\} / \left\{ 1 + (1/k) \frac{\partial}{\partial r} D(r_T, \frac{\pi}{2}, \phi_T) \right\} \quad (10)$$

Equation (10) reduces to Howard's expression for a complex target in the limit when r_T approaches infinity so that $D(r_T, \frac{\pi}{2}, \phi_T)$ is replaced with $\delta(\frac{\pi}{2}, \phi_T)$.

The above results can be extended to the case when elevation error is present. Assume that four coplanar antennas A, B, C, D are utilized to form a tracking antenna system. One pair of antennas, A at (r_A, θ_A, ϕ_A) and B at (r_B, θ_B, ϕ_B) , measure the azimuth angle of the target location relative to the tracker. The other pair of antennas, C at (r_C, θ_C, ϕ_C) and D at (r_D, θ_D, ϕ_D) , measure the elevation angle.

Assume that the antennas C and D are arranged along a line perpendicular to and mutually bisecting with the line connecting the antennas A and B, and the antenna system center T is located at the common midpoint (r_T, θ_T, ϕ_T) of both pairs of antennas. Assume further that the target azimuth angle is determined by phase comparison technique. The azimuth error signal nulling condition leads to :

$$k(r_A - r_B) = D(r_B, \theta_B, \phi_B) - D(r_A, \theta_A, \phi_A) \quad (11)$$

which is just an extension of equation (4) from the $\theta = \pi/2$ plane to a plane containing the target and the antennas A and B. This plane and the direction of the tracking axis cannot be determined until the mechanism for determining the elevation angle is specified. If phase comparison technique is also used to determine the target elevation angle, the elevation error signal nulling condition leads to :

$$k(r_C - r_D) = D(r_D, \theta_D, \phi_D) - D(r_C, \theta_C, \phi_C) \quad (12)$$

Note that equations (11) and (12) are obtained independently and though $\phi_A = \phi_B$ and $\phi_C = \phi_D$, there is no connection between ϕ_A and ϕ_C . Equations (11) and (12) together determine the pointing direction of the tracking axis.

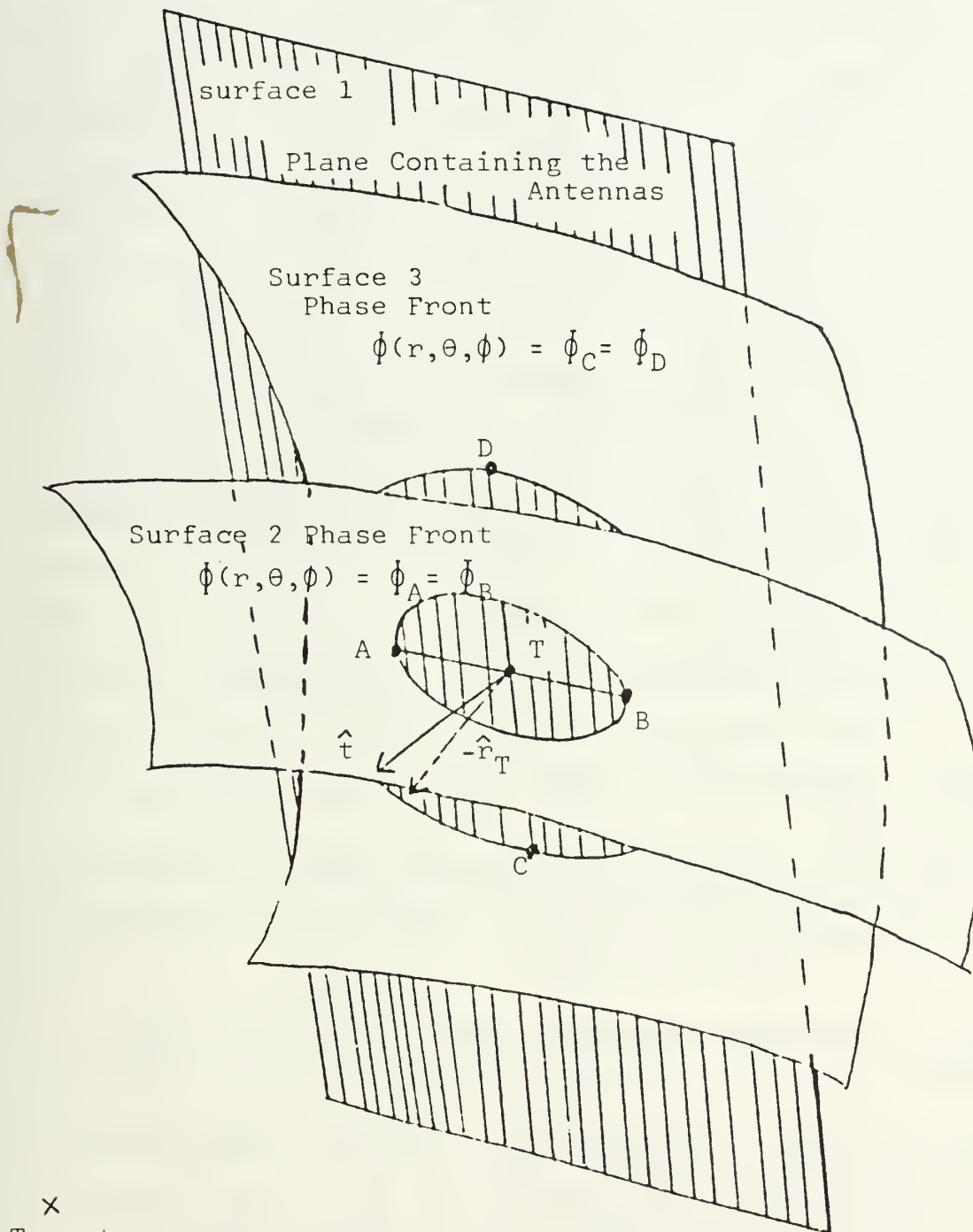


Figure 3.2 Phase Fronts and a Phase Comparison Tracking System.

Figure 3.2 shows a shaded plane, surface 1, which contains the antennas A,B,C,D and the antenna system center T. The tracking axis \hat{t} is normal to surface 1; a phase front surface 2 passes through the antenna pairs A and B; another phase front surface 3 passes through the antenna pairs C and D. Both surface 2 and surface 3 are partially blocked by surface 1, when one looks into the front of the antenna system. If the blocked regions of surface 2 and surface 3 are smooth, the mean value theorem of calculus assures the presence of a normal on each surface in the blocked region which points in the direction of \hat{t} . Since $D(r,\theta,\phi)$ is a slowly varying function of r for large r , together with the assumption that $D(r,\theta,\phi)$ varies slowly over the solid angle extended by the blocked region, the particular normals on surface 2 and surface 3 in the direction of \hat{t} can be approximated by the normal at T of the phase front passing through T (not shown in figure 3.2), with this approximation,

$$\begin{aligned}\hat{t} &= -\nabla\phi(r_T, \theta_T, \phi_T) / |\nabla\phi(r_T, \theta_T, \phi_T)| \\ &= -\{\hat{r} + (1/k)\nabla D(r_T, \theta_T, \phi_T)\} / |\hat{r} + (1/k)\nabla D(r_T, \theta_T, \phi_T)| \quad (13)\end{aligned}$$

For a phase comparison tracking system, equation (13) reduces to equation (10) when there is no error in the elevation angle.

If amplitude comparison technique is used to determine the elevation angle, equation (12) will have to be replaced and equation (13) will not apply.

C. PHASE COMPENSATION EQUATION AND THE ANGULAR TRACKING ERRORS

If the problem of determining the target elevation and azimuth angles are set aside, equations (4), (11), and (12) carry the same message concerning the manner the tracking axis is determined by a phase comparison tracking system.

Take equation (11) as an example. The phase distortion $D(r, \theta, \phi)$ introduces different amounts of phase shift at the two antennas. If the antennas are aligned so that the tracking axis is pointing toward the target direction $-\hat{r}_T$ as shown in figure 3.3 (a), the phases read by the antennas will not be equal and an error signal is generated. The tracking system has to move its axis off this direction to create a difference in the distances traveled by the signals arriving at antennas A and B. This difference in r_A and r_B introduces into the signal a phase difference of $kr_A - kr_B$ which compensates the difference in phase distortions at the antennas as shown in figure 3.3 (b). Thus equation (11) will be referred to as the phase compensation equation.

Let ϕ_e be the angle from the target direction $-\hat{r}_T$ to \hat{t}_a which is in the direction of the projection of the tracking axis \hat{t} on the plane containing the target (origin) and the antennas A and B. Since $k(r_A - r_B)$ can always be approximated by $kd \sin \phi_e$ as long as $r \gg d$, and the antenna system can only measure phase difference to within 2π , the right-hand side of equation (11) will always be a number between $-\pi$ and π . That is, for $r \gg d$,

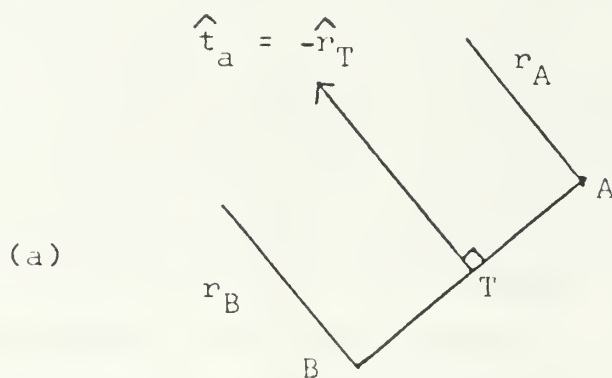
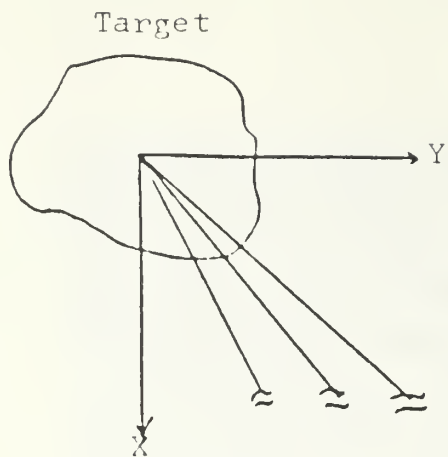
$$kd \sin \phi_e = \{D(r_B, \theta_B, \phi_B) - D(r_A, \theta_A, \phi_A)\} \text{ modulo } 2\pi \quad (14)$$

without any assumption of the smoothness of the phase distortion over a region containing both antennas. For $d > \lambda/2$ equation (14) always has a solution and ϕ_e is bounded :

$$|\phi_e| = \sin(\lambda/2d) \quad (15)$$

Note that if d is increased, $|\phi_e|$ can be limited to within any desired value.

For $d > \lambda$, $k(r_A - r_B)$ may exceed the right-hand-side of equation (11) by 2π . If the tracking axis is pointing along this global error direction, there will be no error signal generated in the tracking system. Figure 3.4 (a) shows the



$$r_A = r_B$$

$$D(r_A, \theta_A, \phi_A) = D(r_B, \theta_B, \phi_B)$$

(b)

$$D(r_B, \theta_B, \phi_B) - D(r_A, \theta_A, \phi_A) = k(r_A - r_B)$$

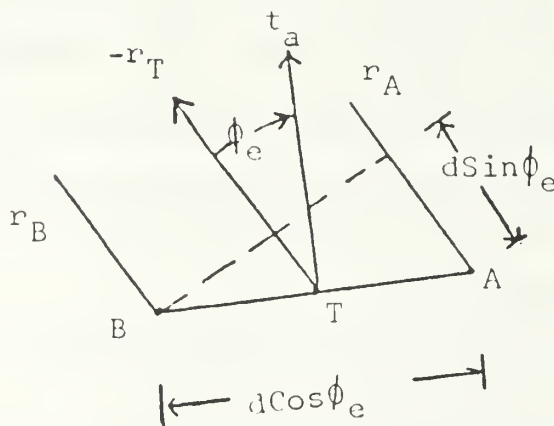
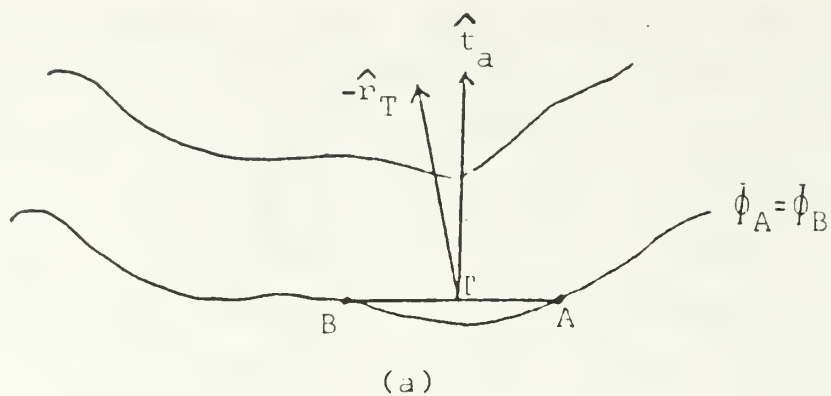


Figure 3.3 Phase Compensation. (a) Error Signal is Generated when the Azimuth Plane Tracking Axis Points to a Target with uneven Phase Distortions at Antennas (b) Uneven Phase Distortions are Compensated by offsetting the Azimuth Plane Tracking Axis.

situation when antennas A and B are on the same phase front. The angular error in this situation will be called a local error and is given by equation (11) or (14). Figure 3.4 (b) shows an example of the global error when the antennas are on different phase fronts with $\phi_A = \phi_B + 2\pi$.

Error signals will be generated by the tracking system to keep the antennas staying on separate phase fronts until target track is lost. Since the angular separation between the global error directions decreases with increasing d , efforts to reduce local error bound will increase the possibility of committing a global error and should be examined carefully.

Figure 3.5 shows the maximum local angular error of a phase comparison system (normalized by π) as a function of antenna separation.



Target x

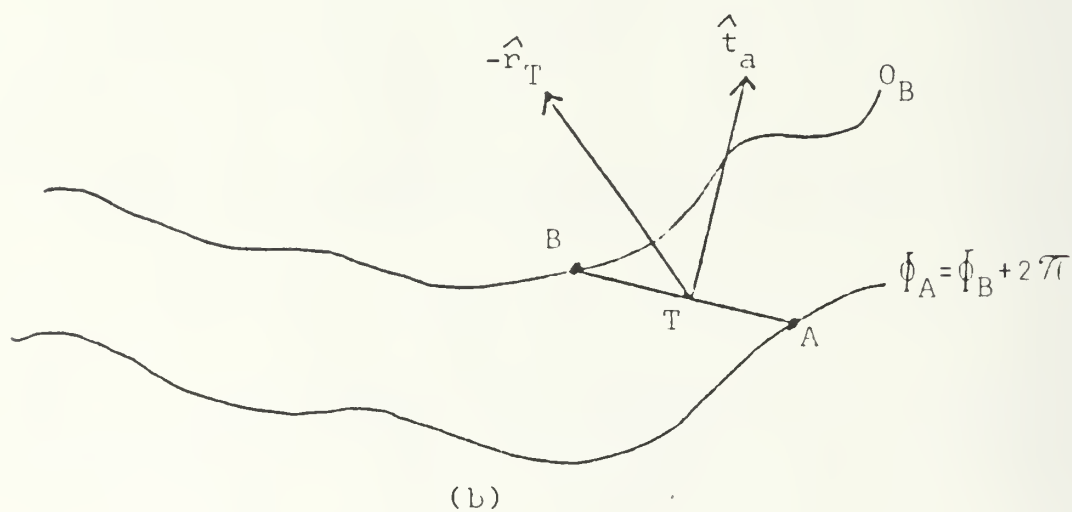


Figure 3.4 Local and Global Angular Tracking Errors.

(a) A Local Angular Tracking Error.

(b) A Global Angular Tracking Error.

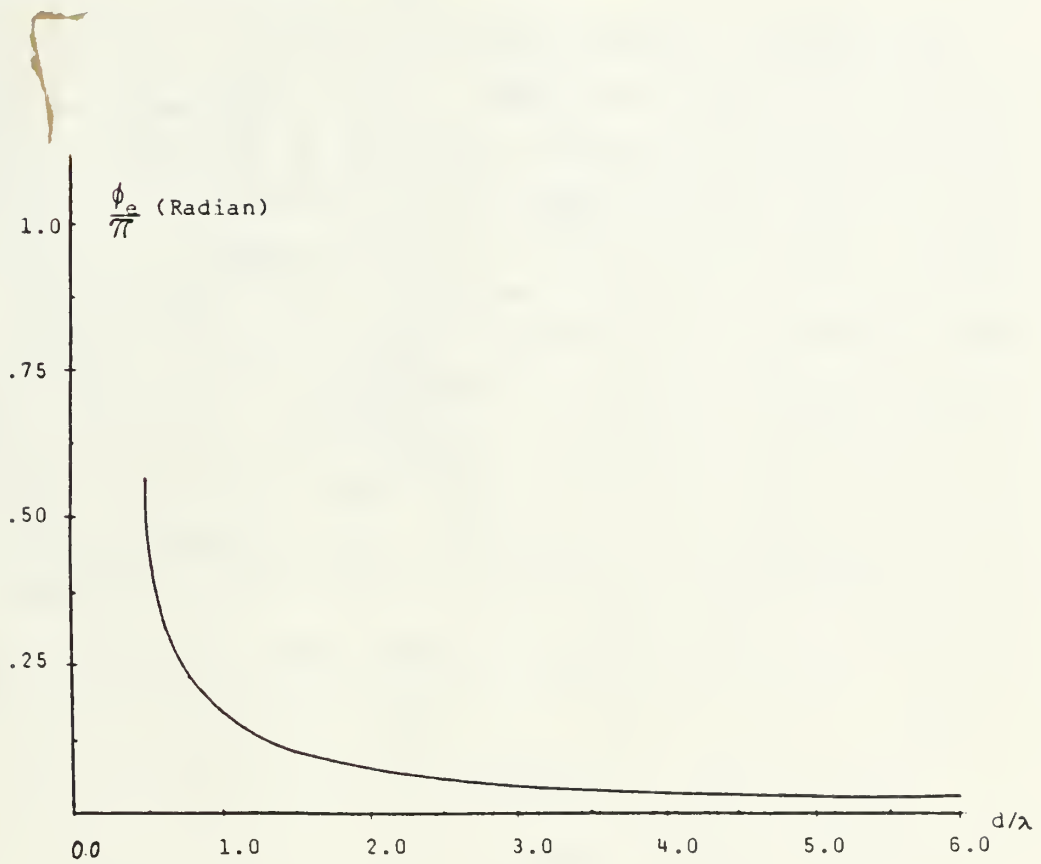


Figure 3.5 Maximum Local Angular Error of a Phase Comparison System as a Function of Antenna Separation.

IV. AZIMUTHAL ANGULAR ERROR FOR A TWO-ELEMENT TARGET

One simple consideration of which the phase distortion $D(r, \theta, \phi)$ and the phase shift $\delta(\theta, \phi)$ possess a singularity across which equations (10) and (13) do not apply is encountered in the phase comparison tracking of a two-element target. The two reflecting elements appear to be two dipole radiators with the same magnitude but 180 degrees out of phase. Assume the tracking antennas are polarized in the z-direction. The target can be modeled as two delta-function current elements located on the y-axis. The current element at $y = 1/2$ has a phase of zero degree while the one at $y = -1/2$ has a phase of -180 degrees. As r approaches infinity, the z-component of the electric field strength becomes

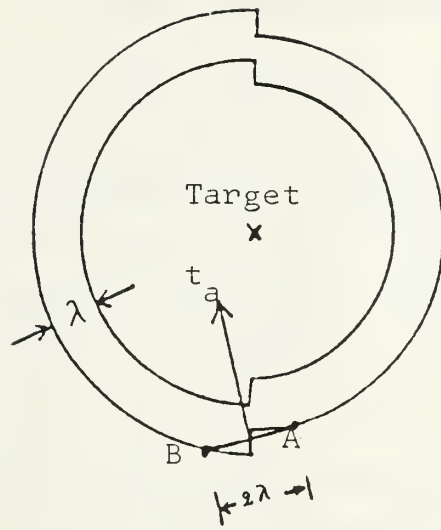
$$E_z(r, \theta, \phi) = (w\mu/2\pi) \sin^2\theta \cdot \sin(kl \cdot \sin\theta \cdot \sin\phi/2) \cdot \{\exp(ikr)\} / r$$

Thus

$$\delta(\theta, \phi) = \begin{cases} 0 & 0 < \phi < \pi \\ -\pi & -\pi \leq \phi < 0 \end{cases}$$

Assume that the tracking system assigns phases to the interval $(-\pi, \pi)$. The phase fronts of the wave and a pair of antennas with separation $d = 2\lambda$ are shown in figure 4.1

Figure 4.2 shows the maximal local error (curve 1), the first global error when the two antennas are separated by 2π (curve 2) and a higher order global error (curve 3) when the antennas are separated by 4π . It can be seen that as d/λ increases, although the local error decreases, the separation between local and global errors also decreases. Thus it is difficult to justify whether it is desirable to increase antenna separation to reduce the local error.



$$|\text{Angular Tracking Error}| = \sin^{-1}(1/4) = 14.48^\circ$$

Figure 4.1 Angular Tracking Error across a Singularity
of the Phase Distortion.

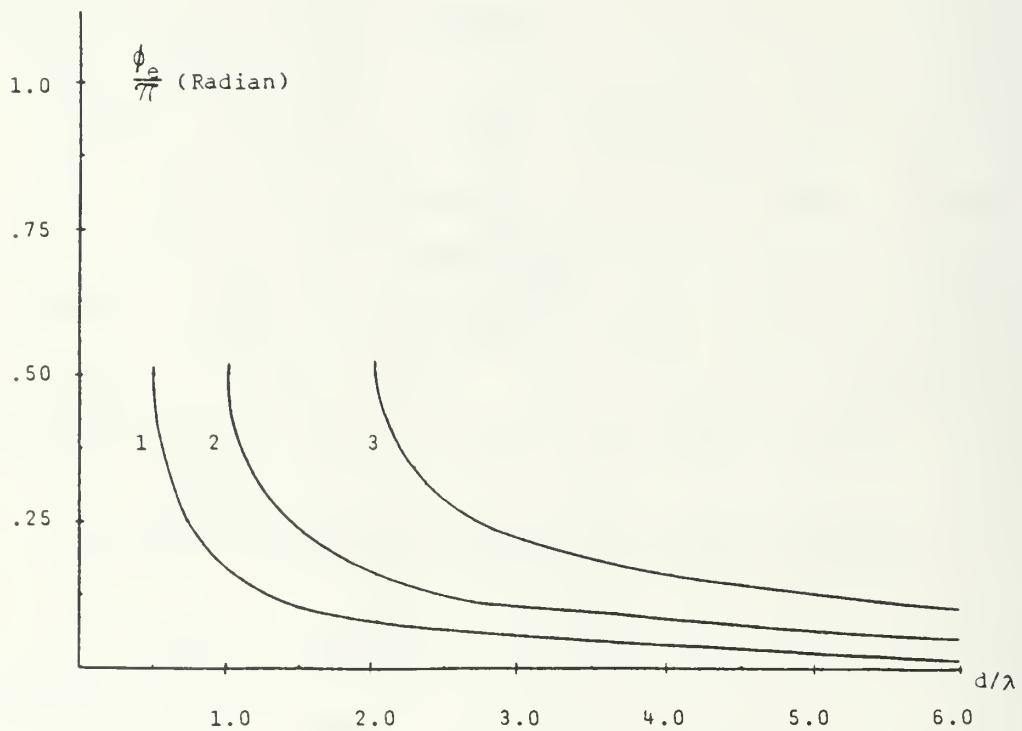


Figure 4.2 Maximum Local Angular Error and Global Error Direction for a Two Element Target as a Function of Antenna Separation.
 1) Maximal Local Error.
 2) First Global Error.
 3) Second Global Error.

V. CONCLUSION AND REMARKS

Howard's phase front distortion technique is proved to be applicable only to a phase comparison tracking system and only if the phase shift of the scattered field of the target is smooth across the solid angle containing the antennas and if the antennas are far from the target so that only the $1/r$ term in the far field is retained. Equations (10) and (13) are extensions of Howard's techniques to closer ranges as long as the target still extends only a negligibly small angle to each of the antennas compared to their beam widths. All these equations are approximations to the phase compensation equation which describes the mechanism of determining the direction of the tracking axis. The phase compensation equation shows that the rate of change of the phase distortion across the finite separation between the antennas governs the tracking axis direction. Rapid variations in the phase distortion between the antennas are averaged out.

Howard's techniques and its extensions, equation (10) and (13), approximate this rate of variation with one over an infinitesimally small distance at the midpoint between the antennas. Hence angular tracking errors will be exaggerated if the phase distortion varies rapidly over the antennas separation, which can happen with a complex target large compared to the wavelength being used.

Through the phase compensation equation, global errors are found to exist for a phase comparison tracking system. For the local error an upper bound is obtained. The antenna separation is an important parameter in all these findings. Its effect on angular error in a phase comparison tracking system should be investigated thoroughly.

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